Team Number:	apmcm24202560
Problem Chosen:	C

Integrated Neural Networks and Optimization for Sustainable Development of China's Pet Food Industry: Analysis and Strategic Planning

This paper addresses the sustainable development of China's pet food industry and provides in-depth research in four aspects: market analysis, demand forecasting, multi-objective optimization and strategy formulation.

For question one, we used **time series decomposition** and studied the development of China's pet industry in the past five years by pet species, successfully identified its long-term trend as well as cyclical characteristics, and **combined the LSTM model with the Attention mechanism** to forecast the development of China's pet industry in the next three years, and **the prediction result error showed that MSE=0.0136,MSLE=0.0012**,. It proves the accuracy of the prediction.

For question two, we **combined LSTM, CNN and Attention mechanism** to construct a global pet food volume demand prediction model, which successfully captured the complex long-term trend and short-term fluctuations, and its prediction results were highly consistent with the actual data (in which **the error analysis showed: MSE=0.0127,MSLE=0.0107**), which proved the accuracy of the prediction.

For question three, we further predicted the production and export of pet food in China in the next three years by following the neural network model constructed in question two, and **the prediction model is still accurate.**

For question four, we optimized the balance of economic benefits, environmental impacts and resource consumption by **using a multi-objective optimization model (NSGA-II)**, and proposed a production and export strategy suitable for China's pet food industry, and **the model's efficiency and superiority were verified by sensitivity analysis**. Meanwhile, a sustainable development strategy is formulated, and the model is able to dynamically respond to policy changes and external shocks with high applicability and robustness.

The research in this paper provides scientific decision-making support for China's pet food industry and provides a reference for realizing the synergistic development of economy, society and environment.

Keywords: Pet food industry LSTM neural network CNN Convolution time series forecast NSGA-II algorithm Sustainable development strategies

Contents

I. Introduction

In recent years, the Chinese pet food market has undergone a significant evolution, characterized by rapid growth in market size, category diversification and regional concentration. The aim of this paper is to analyze the factors driving this growth and provide a structured approach to understanding and forecasting future trends for sustainable development of the industry. The visual analysis provided in this paper highlights key aspects of this growth, including geographic concentration, time trends, and cross-category dynamics.

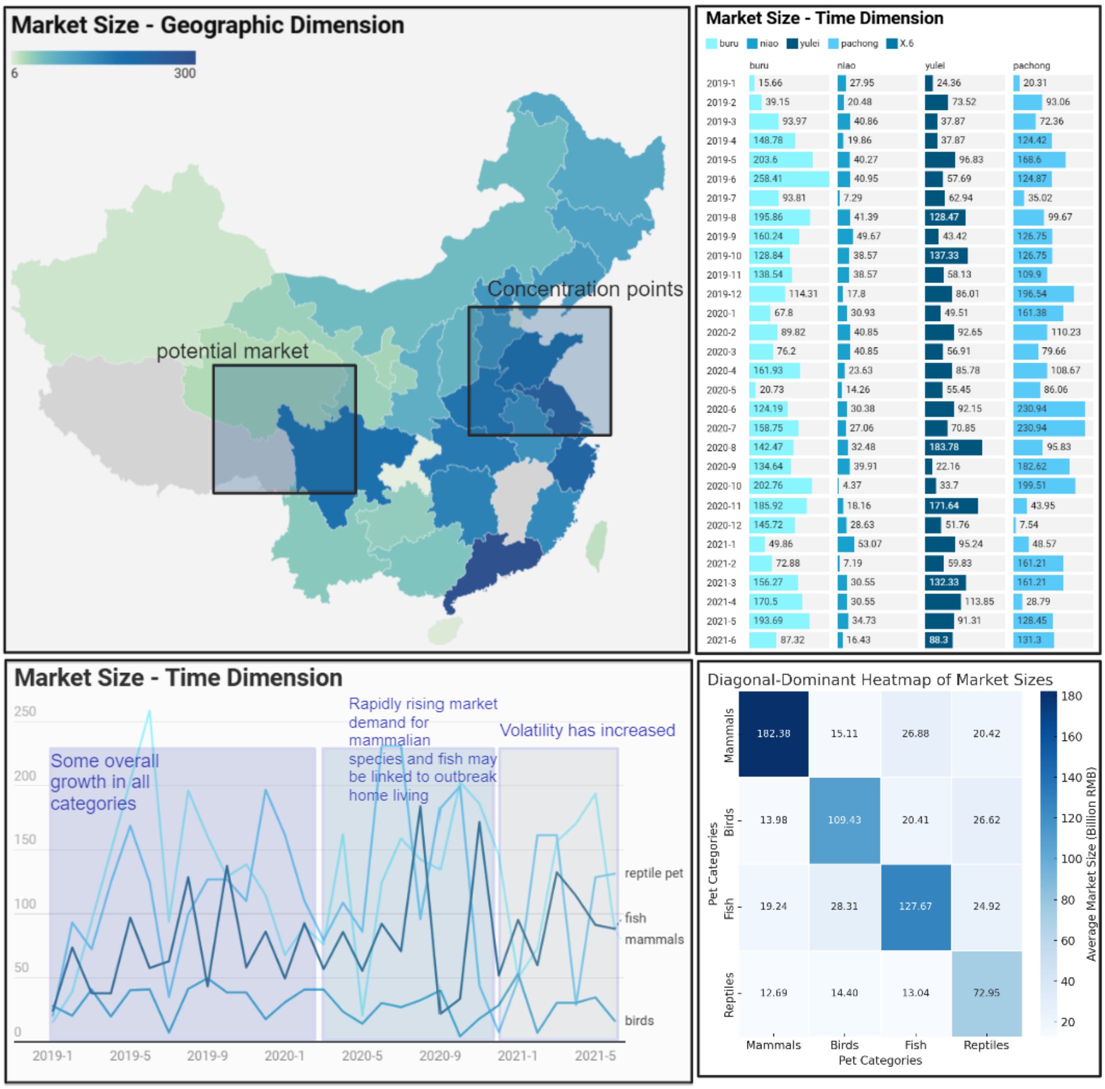

Figure 1 Comprehensive situation of China's pet market

II. Assumptions and Justifications

We assume **a linear relationship between market size $M_{i,t}$ and consumption levels** (e.g., expenditures on daily necessities, healthcare expenditures, etc.), **policy factors, and control variables** :

- Changes in market size can be thought of as the additive effect of consumer expenditures, with expenditures on daily necessities, medical care, beauty care, and insurance collectively determining market demand.
- The linear relationship assumption can clearly express the direct effect of each type of consumer expenditure on market size.
- The impact of policy support (e.g., subsidies) or restrictions (e.g., regulations) on market size can usually be quantified through proportional changes and therefore has an approximately linear relationship with market size.

III. Notations

Table 1 Symbols and Their Descriptions

Symbol	Description
P_t	Monthly production volume at time t (in 10,000 tons)
E_t	Monthly export value at time t (in billion USD)
C_t	Carbon emissions at time t (in tons)
R_t	Energy consumption at time t (in megawatt-hours)
β_c	Carbon emission coefficient per unit of production
β_r	Energy consumption coefficient per unit of production
S_t	Domestic market share at time t
T_t	Long-term trend component of time series
S_t	Seasonal component of time series
R_t	Residual component of time series
y_t	Observed value at time t in the time series
α, β	Weights for domestic sales and exports in economic objectives
Z_1	Objective function for maximizing economic benefits
Z_2	Objective function for minimizing environmental impact
Z_3	Objective function for minimizing resource consumption
H_{LSTM}	Hidden states output by the LSTM layer
H_{CNN}	Feature maps extracted by the CNN layer
z_{fusion}	Combined feature vector from LSTM, CNN, and Attention layers
$\hat{y}$	Predicted demand for the next 36 months
η_c	Probability parameter for simulated binary crossover in NSGA-II
d_i	Crowding distance for solution i
$f_i(x)$	i-th objective function for solution x

IV. Modeling and solving of Question I

4.1 Data Description

In order to better understand the development trend of China's pet industry and make accurate predictions, this model study collects relevant data from multiple dimensions and constructs a high-quality dataset through classification and statistical methods.

The main objectives of data collection include:

1. **Market size analysis**: To analyze the distribution of market demand in different regions by counting the number of pets in each province nationwide.
2. **Species classification and characterization study**: to count the number of major pet species in China and categorize them into mammals, birds, fish and reptile pets to reveal structural differences in the market.
3. **Time series study**: analyze the dynamics of the pet population and the market in each category between 2019 and 2023.
4. **Data forecasting modeling**: to provide basic data support for future market size forecasting and strategy development.

4.1.1 *Data Set Structure*

The dataset for this study is divided into two parts, **spatial dimension data** (number of pets by province) and **temporal dimension data** (changes in the number of pet categories by month).

Table 2 Spatial Distribution of Pet Categories in China

Province	Mammals (10K)	Birds (10K)	Fish (10K)	Reptiles (10K)
Beijing	150	50	30	10
Shanghai	180	60	40	12
Guangdong	300	100	80	25
Zhejiang	220	70	60	18
Sichuan	200	65	55	17

4.1.2 *Classification of Pet Species*

Based on biological characteristics, common pet ownership practices and market relevance, and for research convenience, we categorized pet species into four main groups - **mammals, birds, fish and reptiles**. This categorization process involves three main steps:

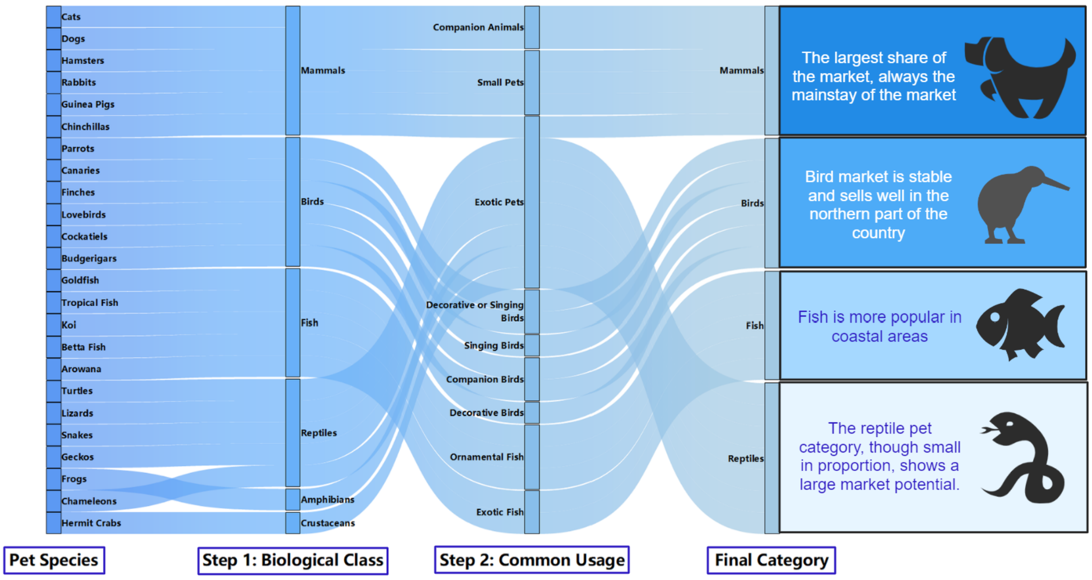

Figure 2 Classification of Pet Species

1. **Biological Classification (Step 1):** Categorizing each species based on its **biological taxonomy**, such as mammals, birds, fish, amphibians, or reptiles.
2. **Usage-Based Classification (Step 2):** Further refining the categorization based on their **typical uses or roles in pet-keeping** (e.g., companion animals, ornamental pets, exotic pets).
3. **Final Assignment (Step 3):** Mapping the refined classifications into the four broader categories: Mammals, Birds, Fish, and Reptiles, to align with market and research needs.

4.2 The Establishment of Model I

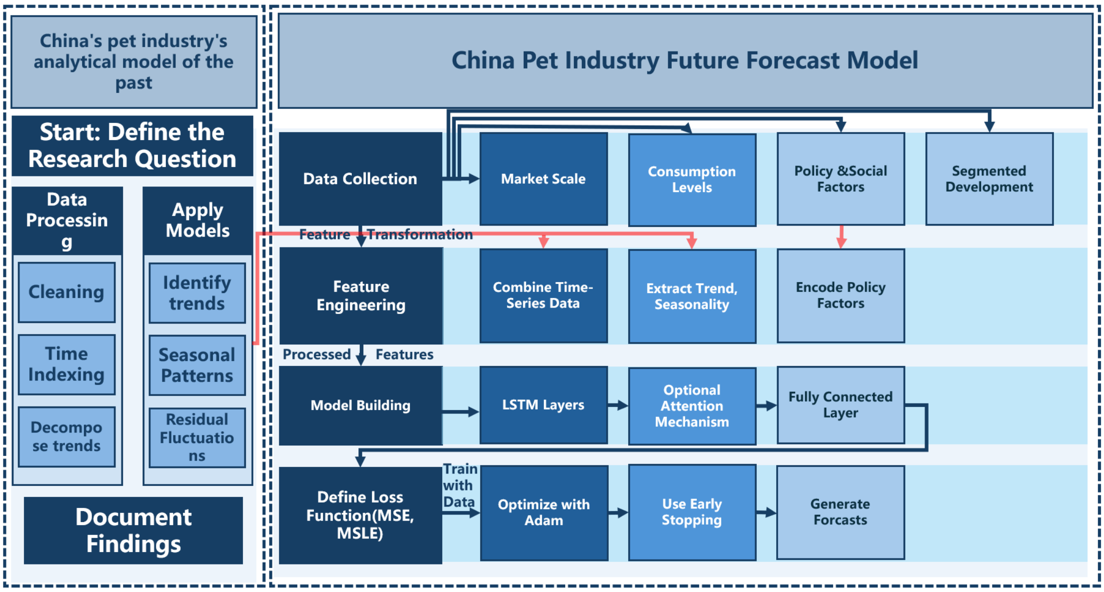

Figure 3 Flowchart for Model I

4.2.1 *Analysis of the development of the pet industry in China*

Time series decomposition:

We look at the observed data by splitting it into several components: **Trend, Seasonality, and Residual**, with the goal of extracting long-term trends, cyclical fluctuations, and stochastic fluctuations in the time series, respectively.

We chose **the multiplicative model** : for time series in which seasonal fluctuations are amplified by changes in trend:

$$Y_t = T_t \cdot S_t \cdot R_t$$

where:

- Y_t denotes observations at time t;
- T_t is the **trend component**, reflecting long-term changes;
- S_t is the **seasonal component**, reflecting cyclical fluctuations;
- R_t is the **residual component**, which represents unexplained random fluctuations.

Where the trend $T_{t,i}$ are modeled as a combination of linear and nonlinear:

$$T_{t,i} = \beta_{i,0} + \beta_{i,1} t + \beta_{i,2} t^2$$

Among them:

- $\beta_{i,0}$: initial value.
- $\beta_{i,1}$: linear growth rate.
- $\beta_{i,2}$: non-linear growth rate.

We used a periodic function to model the representation of seasonality $S_{t,i}$ in the time series:

$$S_{t,i} = A_i \sin\left(\frac{2\pi t}{P_i} + \phi_i\right)$$

Among them:

- A_i: amplitude, indicating the magnitude of fluctuation.
- P_i: period length (e.g. 12 months).
- ϕ_i: phase offset, indicating the starting point of the fluctuation.

For phenomena such as noise burrs due to other conditions, we assume the residuals $R_{t,i}$ to be normally distributed with a mean of zero:

$$R_{t,i} \sim N(0, \sigma_i^2)$$

We assume that the global market size is G_t, and the European and U.S. market shares are E_t and U_t, respectively:

$$E_t = \alpha_e G_t, \quad U_t = \alpha_u G_t \quad \alpha_e + \alpha_u \leq 1$$

We fit the trend and seasonality models using the least squares (OLS) method, setting the objective function as:

$$\min \sum_{t=1}^{N} \sum_{i=1}^{4} \left(y_{t,i} - (T_{t,i} + S_{t,i})\right)^2$$

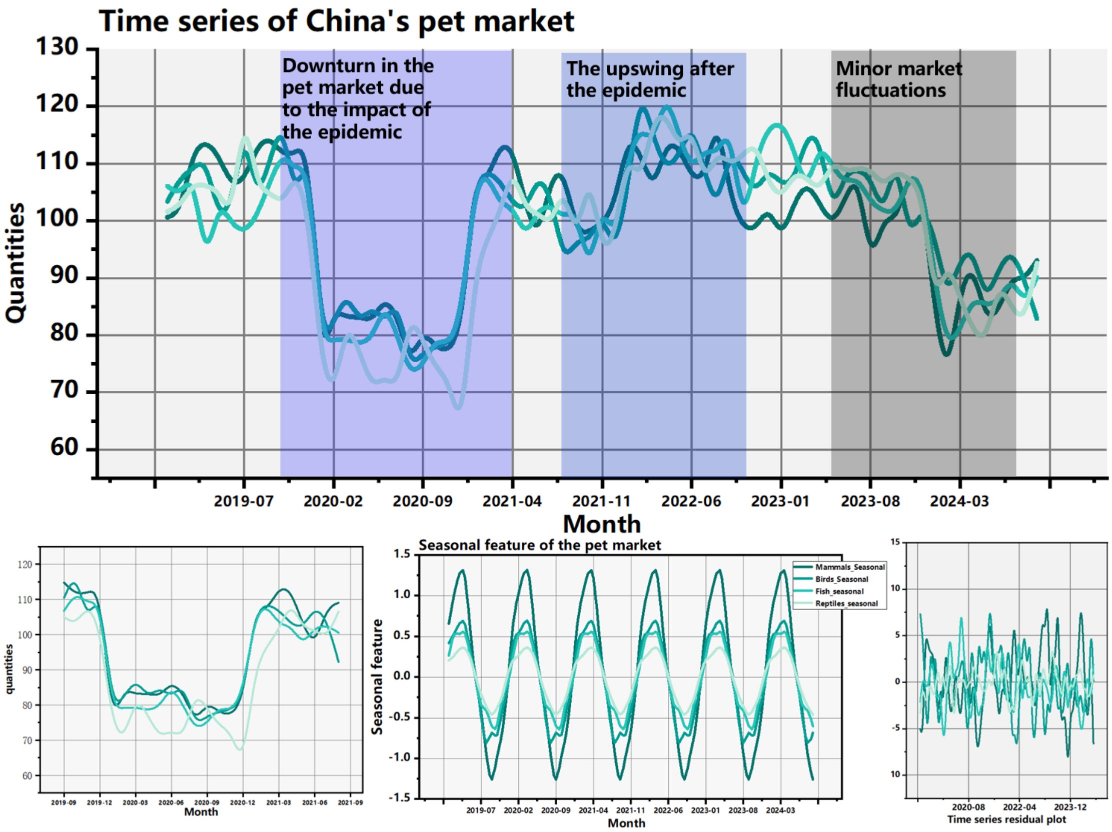

Figure 4 Time series analysis of China's pet industry over the past five years

After solving the least squares method, we obtain the decomposition result in the above figure, which we analyze in the following ways:

Overall Trend: The overall trend illustrates significant changes in the pet market over the past five years. A noticeable **downturn** occurred during the early phase of the pandemic in 2020, as shown by a decline in Y_t for all pet categories. This was caused by disruptions in supply chains and reduced consumer spending. A recovery phase followed between late 2021 and early 2022, marked by a steady increase in T_t, particularly for mammals and birds. By 2023, the market stabilized with minor fluctuations, indicating a maturing industry.

Seasonality: Seasonal patterns vary across pet categories. Mammals (S_t) exhibit the most pronounced seasonality, with demand peaking during holidays (e.g., Chinese New Year) and spring. Birds show moderate seasonal variation, with slightly increased demand in spring and early summer due to breeding cycles. Fish and reptiles have relatively stable seasonal patterns, with minimal fluctuations throughout the year. These seasonal features suggest specific consumer behaviors tied to cultural and biological factors.

Residuals: The residuals (R_t) represent random fluctuations that are not explained by trend or seasonality. During the pandemic (2020), residuals increased significantly, indicating unpredictable market dynamics caused by external shocks. Post-recovery, R_t values decreased, showing reduced volatility and a more stable market environment. Residual analysis helps identify periods of market disruption and assess the impact of unforeseen factors.

4.2.2 *Forecasting model for China's pet industry*

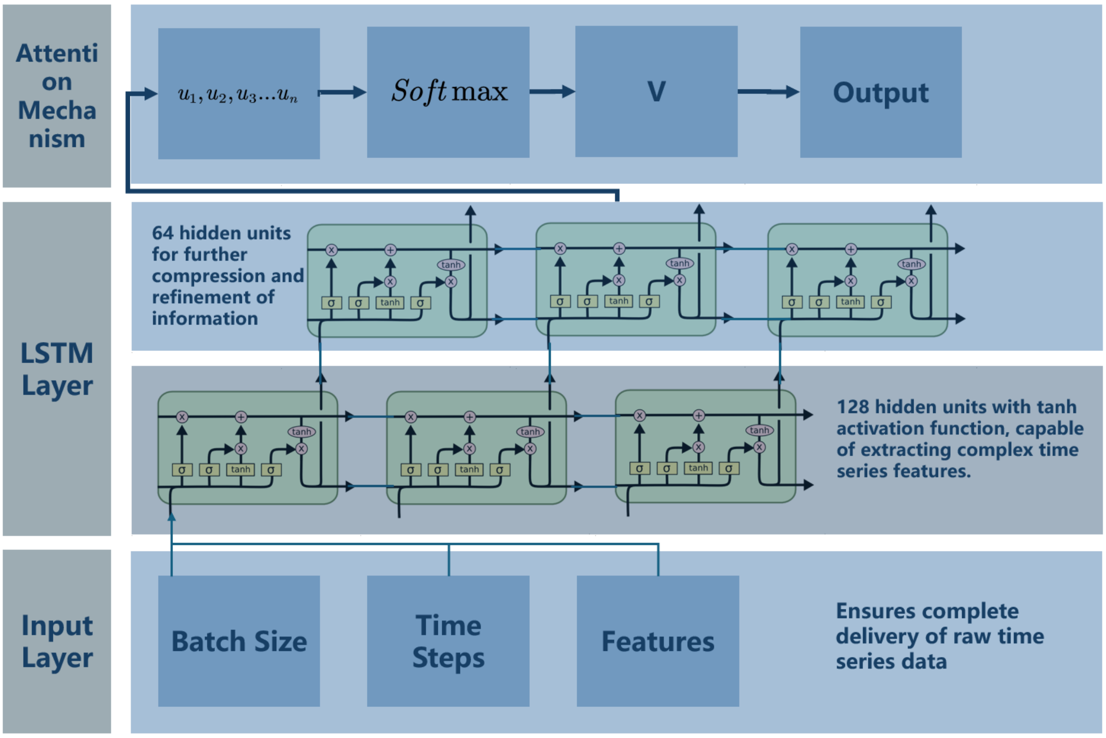

Figure 5 LSTM prediction model structure

The model designed in this study mainly consists of an input layer, an LSTM layer, a fully-connected layer, and an output layer, aiming to **capture the time-series characteristics of the pet market and construct a forecast model of future market size based on multiple factors**.

I Input Layer:

The input layer is responsible for receiving multidimensional time series data as the main input to the model, and its shape is defined as:

$$(batch\ size, time\ steps, features)$$

- **Batch size**: the number of samples for each training input.
- **Time steps**: the length of the input time series.
- **Features**: the dimensions of the input features, each feature corresponds to the value of four major factors.

This layer ensures the complete delivery of the original time series data and provides the full amount of information input to the subsequent LSTM layer.

II LSTM Layer:

The LSTM (Long Short-Term Memory) layer is the core of the model, which is used to capture the long-term dependencies and short-term dynamic features of the time series. In this study, multiple layers of stacked LSTM cells are used to gradually extract the hidden structural information in the time series.

Structural design:

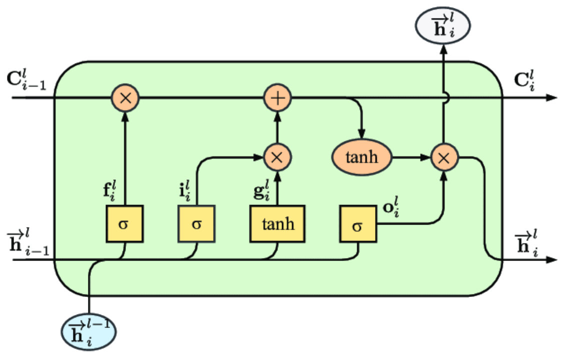

Figure 6 Structure of a single LSTM cell

Layer 1: contains 128 hidden units with an activation function of tanh, capable of extracting complex time series features.

Layer 2: contains 64 hidden units, which are used to further compress and refine the information.

$$h_t, c_t = \text{LSTM}(X_t, h_{t-1}, c_{t-1})$$

where:

- X_t is the input for time step t;
- h_t is the hidden state;
- c_t is the cell state;

The LSTM layer is the core computational function of the long and short term memory cell.

III Attention mechanism:

To **enhance the model's ability to capture multidimensional input features**, this study introduces an attention layer in the model. The attention mechanism generates weight coefficients by learning the importance of different input features, thus improving the interpretability of the model:

$$\alpha_t = \frac{\exp(e_t)}{\sum_{t=1}^{T} \exp(e_t)}, \quad e_t = \text{score}(h_t, X_t)$$

where:

- α_t is the attentional weight of time step t;
- score denotes the importance of computing the current time step.

IV Fully Connected Layer:

The fully connected layer receives the output from the LSTM layer and is further compressed to the target dimension by the neural network. The output of this layer is shaped as:

$$(\text{batch size, forecast horizon})$$

V Output Layer:

The output layer is responsible for generating the final forecasts, i.e., the time-series predictions of the pet market size for each category.

4.3 Model prediction results and analysis

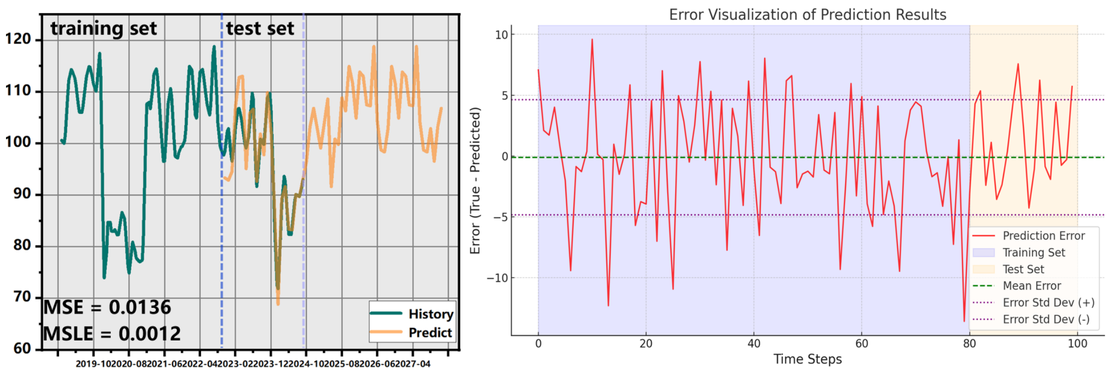

Figure 7 Plot of prediction results against error

We take the prediction results of mammalian pets as an example and analyze the prediction results and errors in detail:

Test Set component (Test Set):

On the Test Set, the predicted values were able to continue the trend of the Training Set with a relatively high degree of accuracy, and responded reasonably well to the more volatile time points (e.g., early 2023).

This suggests that the model has some generalization ability and is able to cope with new data in the Test Set.

Evaluation Metrics:

Mean Square Error (MSE) of 0.0136 and **Mean Square Logarithmic Error (MSLE) of** 0.0012, suggesting that the model performs well on the continuous value prediction task with low error.

V. Modeling and solving of Question II

5.1 Data Description

To address this issue, we have collected five years of market data on the global pet industry, categorized by country and region. The data spans continents and markets, reflecting pet market dynamics during periods of stability and disruption, such as the COVID-19 pandemic.

The main components of the data include:

- **Temporal Coverage**: January 2019 to September 2024, with monthly demand figures.
- **Regional Scope**: Data covers major markets, including the United States, China, Europe, South America, Africa, and Asia-Pacific regions.
- **Granularity**: Pet food demand is quantified by country, measured in terms of quantity demanded.

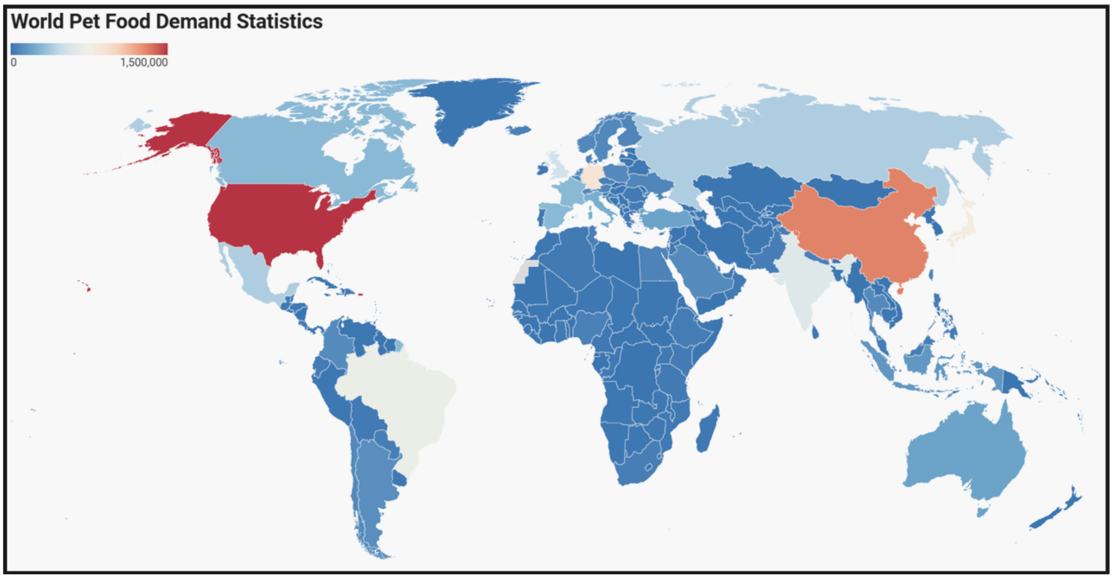

Figure 8 World Pet Food Demand Statistics(2023)

The graph above shows the global distribution of pet food demand. Countries are colored according to their level of demand:

- **High Demand Areas:** The United States and China have the highest demand, at more than 1.5 million tons per year. These countries dominate the global pet food market due to high pet ownership and a mature consumer market.
- **Medium Demand:** European countries such as Belgium and developing markets such as Brazil and India have a steady to moderate demand for pet food.
- **Low Demand:** Low demand in parts of Africa and Southeast Asia, including Kenya and Vietnam. However, these regions show potential for growth as pet ownership increases.

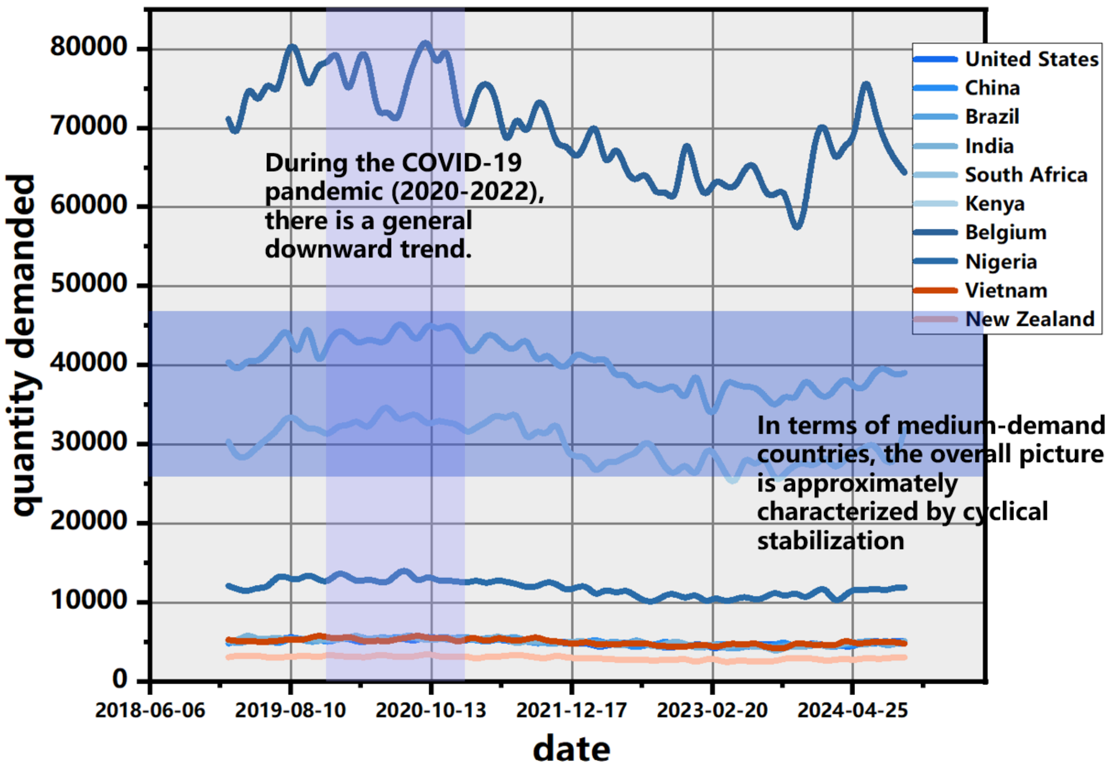

Figure 9 World Pet Food Demand Trend(2018-2024)

The figure above provides a time-series representation of pet food demand across selected countries.

5.2 The Establishment of Model II

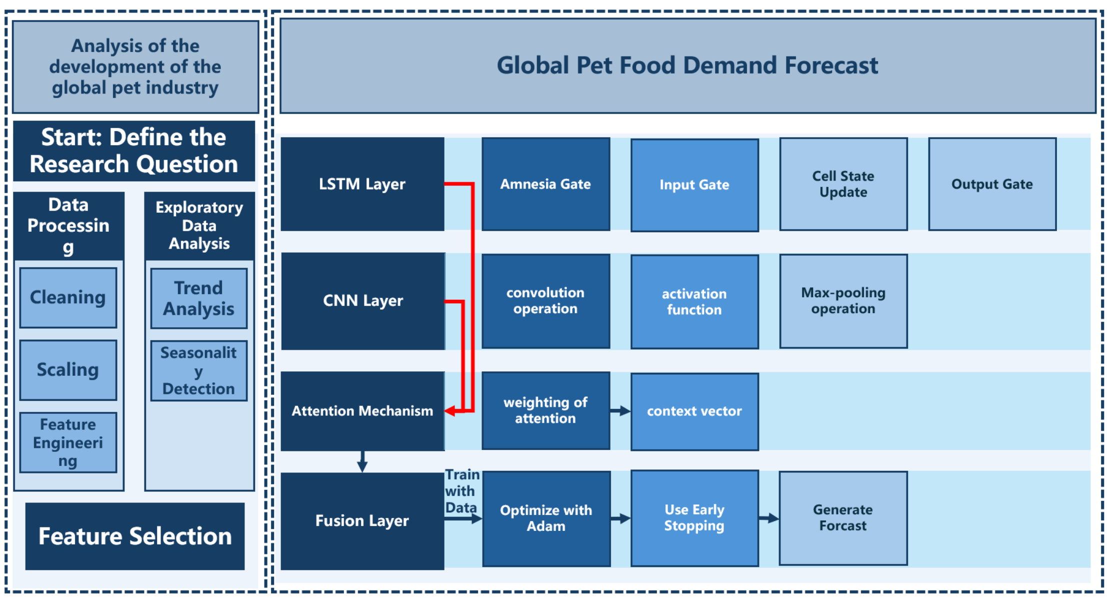

Figure 10 Flowchart for analyzing predictions for model 2

5.2.1 *Analysis of the development of the global pet industry*

We break down the growth of the pet industry by pet type into three parts:

1. **Trend Analysis**: Identifies the long-term growth trajectory of each pet type.
2. **Seasonal Analysis**: Explores cyclical fluctuations in demand and ownership.
3. **Regional Contribution**: Assesses the geographic distribution of market growth.

Because of the more pronounced seasonality of the data, unlike the analysis of the pet market in China, this time we used an additive decomposition model. The mathematical formulation of the analysis is expressed as:

$$y_{t,i,r} = T_{t,i} + S_{t,i} + R_{t,i}$$

Where:

- $y_{t,i,r}$: Market size for pet type i in region r at time t.
- $T_{t,i}$: Long-term trend for pet type i.
- $S_{t,i}$: Seasonal variation for pet type i.
- $R_{t,i}$: Residual noise capturing irregularities.

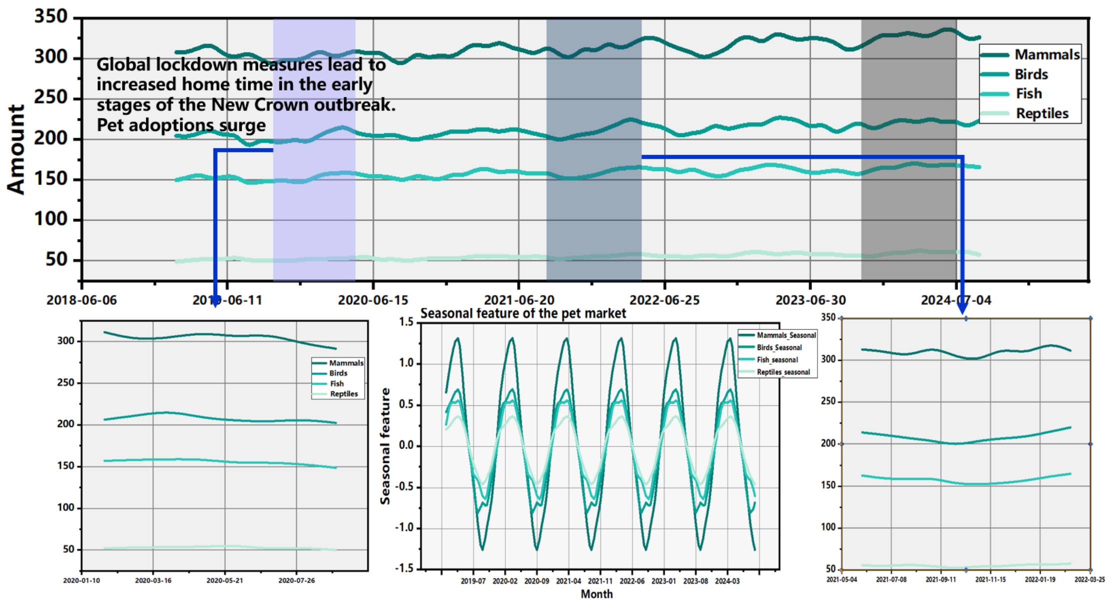

Figure 11 Time Series Breakdown of the Global Pet Industry Situation

Trend Analysis

- **Mammals**: Exhibit a clear and dominant upward trend, reflecting their primary role in the pet market. Growth accelerates during the early pandemic (2020–2022) as pet adoptions surged globally due to lockdown measures.

- **Birds and Fish**: Show steady but slower growth over the analysis period. Their popularity increased slightly during the pandemic, likely due to their suitability for urban living.
- **Reptiles**: Exhibit the smallest upward trend, reflecting their niche appeal but consistent growth among specific demographics.

Seasonal Component

- **Mammals**: Strong seasonal fluctuations, peaking during holiday seasons (e.g., Q4), driven by increased gifting and spending on pets.
- **Birds**: Moderate seasonality, with small peaks likely associated with spring breeding periods.
- **Fish and Reptiles**: Minimal seasonal variation, reflecting steady demand throughout the year.

5.2.2 *Global Pet Food Demand Modeling*

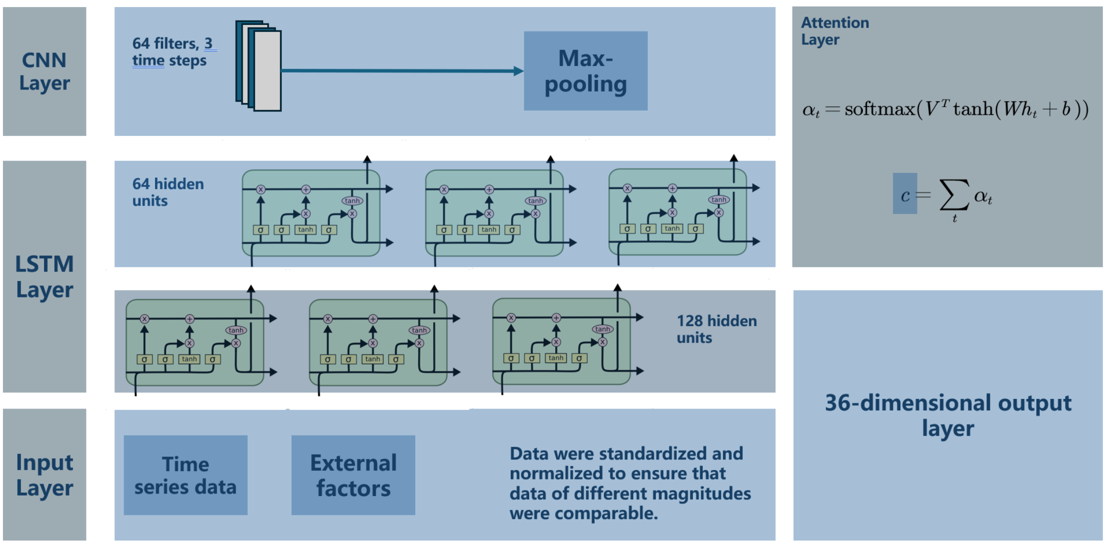

Figure 12 Global Pet Food Demand Forecast Model Structure

In order to accurately forecast global pet food demand over the next three years (36 months), we constructed a hybrid model architecture combining LSTM (Long Short-Term Memory Network), CNN (Convolutional Neural Network) and Attention (Attention Mechanism). The model aims to capture long-term trends, local variations, and contributions from key time steps from historical time series, and incorporates external influences for comprehensive forecasting.

Input Layer: The input data includes:

Historical time series data:

- Total global pet food demand x_t^{total}
- Demand for each type of pet food $x_{t,i}$

External influences:

- Macroeconomic data (e.g., GDP growth rate, inflation rate, urbanization rate).
- Seasonal variables (e.g., months, holiday markers).

Input Format:

$$X_{\text{input}} = \{x_{t-L+1}, x_{t-L+2}, \ldots, x_t\}$$

where L is the length of the time window, e.g., the last 12 months of data.

LSTM layer: LSTM is used to capture long-term dependencies in time series.

Structure:

- **Input Sequence:** A historical time series of length L, denoted as $\{x_{t-L+1}, \ldots, x_t\}$, where x_t represents the input at time step t.

LSTM Unit Computations:

The LSTM unit consists of four key components: **Forget Gate, Input Gate, Cell State Update, and Output Gate**. The mathematical computations for each component are detailed below:

1. Forget Gate:

The forget gate decides which information from the previous cell state C_{t-1} to retain. It is defined as:

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

where:

- f_t: Forget gate activation vector.
- W_f: Weight matrix for the forget gate.
- b_f: Bias vector for the forget gate.
- h_{t-1}: Hidden state from the previous time step.
- x_t: Current input vector.
- σ: Sigmoid activation function.

2. Input Gate:

The input gate determines which new information to store in the cell state. It is computed as:

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

where:

- i_t: Input gate activation vector.
- $\tilde{C}_t$: Candidate values for the cell state.
- W_i, W_C: Weight matrices for the input gate and cell state update.
- b_i, b_C: Bias vectors for the input gate and cell state update.
- tanh: Hyperbolic tangent activation function.

3. Cell State Update:

The new cell state is updated as:

$$C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t$$

where:

- C_t: Updated cell state.
- C_{t-1}: Previous cell state.
- $\cdot$: Element-wise multiplication.

4. Output Gate:

The output gate decides the next hidden state h_t, which is used for predictions or as input to the next time step:

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t \cdot \tanh(C_t)$$

where:

- o_t: Output gate activation vector.
- W_o: Weight matrix for the output gate.
- b_o: Bias vector for the output gate.
- h_t: Current hidden state.

Output

The LSTM outputs a sequence of hidden states:

$$H_{\text{LSTM}} = \{h_{t-L+1}, \ldots, h_t\}$$

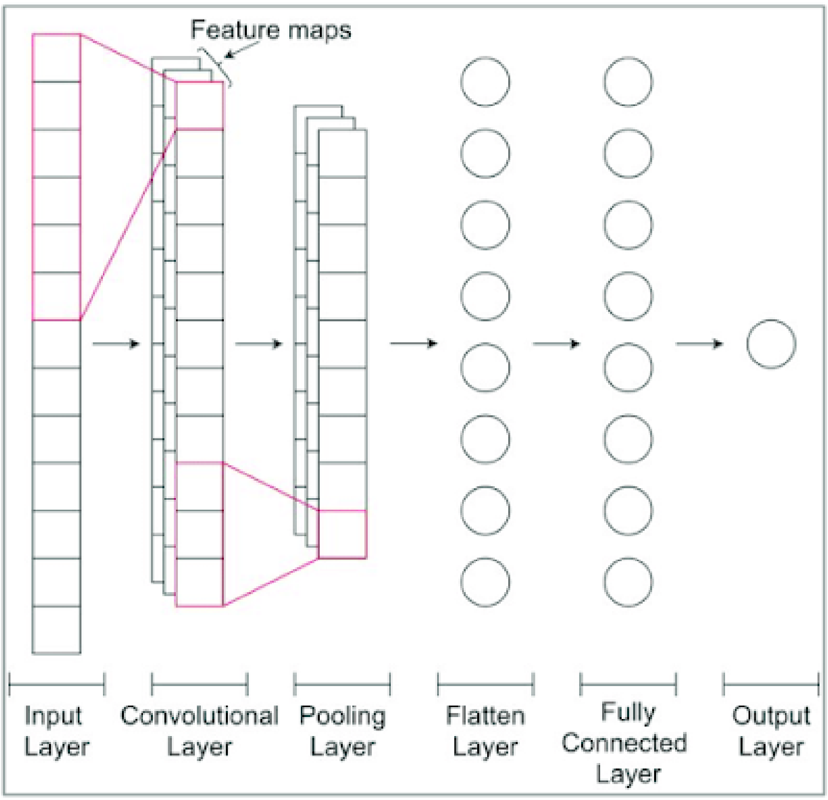

Figure 13 1D-CNN architecture

CNN Layer: The Convolutional Neural Network (CNN) is used to extract short-term patterns (e.g., seasonal fluctuations) and local features from the time series data.

Convolution Computation

1. Convolution Operation:

The convolution operation calculates local features using a sliding window of size K:

$$z_t = \sum_{k=1}^{K} w_k \cdot x_{t-k+1} + b$$

where:

- K: Size of the convolution kernel, determining the temporal window range.
- w_k: Weights of the convolution kernel.
- b: Bias term.
- x_t: Input data at time step t.

2. Activation Function:

The output of the convolution operation is passed through a ReLU activation function:

$$a_t = \mathrm{ReLU}(z_t)$$

where:

- a_t: Activated feature map.
- $\text{ReLU}(z_t) = \max(0, z_t)$: The Rectified Linear Unit (ReLU) function.

3. Max Pooling:

To reduce the feature dimensionality and focus on the most significant features, max pooling is applied:

$$a_{\text{pool}} = \max(a_t)$$

where:

- a_{pool}: The maximum value within the pooling window.

Output

The CNN extracts a sequence of local features:

$$H_{\text{CNN}} = \{a_{\text{pool},t-L+1}, \ldots, a_{\text{pool},t}\}$$

Fusion Layer and Output Layer: **Fusion Layer:** The fusion layer combines features extracted from the LSTM, CNN, and Attention layers into a high-dimensional feature vector for further processing.

Fusion Formula: The fusion operation concatenates the outputs of the LSTM layer (H_{LSTM}), CNN layer (H_{CNN}), and Attention layer (c) as:

$$z_{\text{fusion}} = \text{concat}(H_{\text{LSTM}}, H_{\text{CNN}}, c)$$

Dense Layer: The concatenated feature vector is passed through a dense layer with a non-linear activation function (ReLU) to compress and refine the features:

$$z = \text{ReLU}(W_{\text{dense}} \cdot z_{\text{fusion}} + b_{\text{dense}})$$

where:

- W_{dense}: Weight matrix of the dense layer.
- b_{dense}: Bias vector of the dense layer.
- z: Refined feature vector after the dense layer.

Output Layer: The output layer predicts the global pet food demand for the next 36 months.

Output Formula: The output is computed as:

$$\hat{y} = W_{\text{output}} \cdot z + b_{\text{output}}$$

where:

- $\hat{y}$: Predicted demand vector of size 36, corresponding to the demand for the next 36 months.
- W_{output}: Weight matrix of the output layer.
- b_{output}: Bias vector of the output layer.

5.3 Model prediction results and analysis

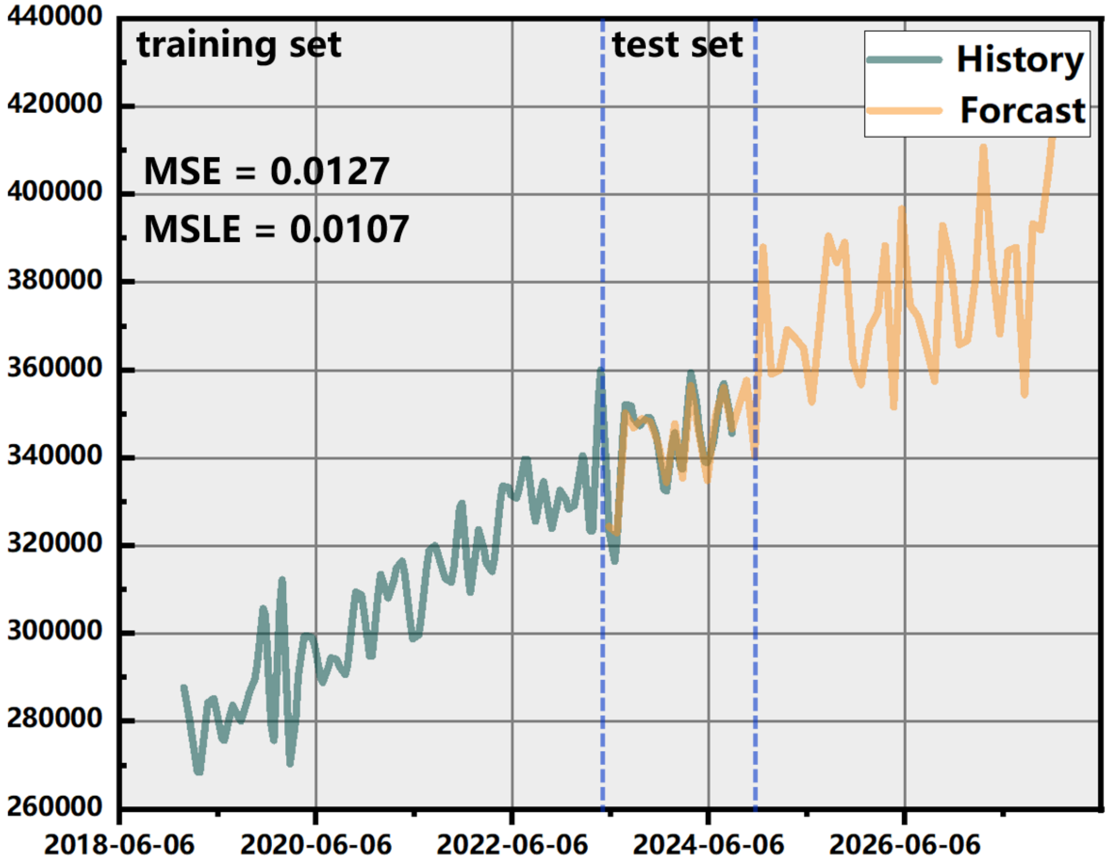

Figure 14 Global Pet Food Demand Forecast Results

After the prediction of the model, we get the prediction as shown in Fig:

MSE (Mean Squared Error): 0.0127

MSE measures the average squared difference between the actual and predicted values in the training set. A low MSE indicates that the model is fitting the training data well.

MSLE (Mean Squared Logarithmic Error): 0.0107

MSLE evaluates the ratio of actual to predicted values. It penalizes underestimations less severely than overestimations and is well-suited for data with exponential growth trends.

VI. Modeling and solving of Question III

6.1 Data Description

The dataset utilized in this study is derived from a combination of historical records and market reports focusing on China's pet food industry and the global pet food market trends. Key aspects of the data include:

- **Export Data**: Historical export values (in billion USD) and volumes (in tons) of China's pet food, segmented by major trading partners and regions.
- **Production Data**: Historical trends in China's pet food production volumes (in 10,000 tons).
- **Market Distribution**: Breakdown of export destinations by market share, highlighting contributions from countries such as the United States, Germany, Japan, and others.

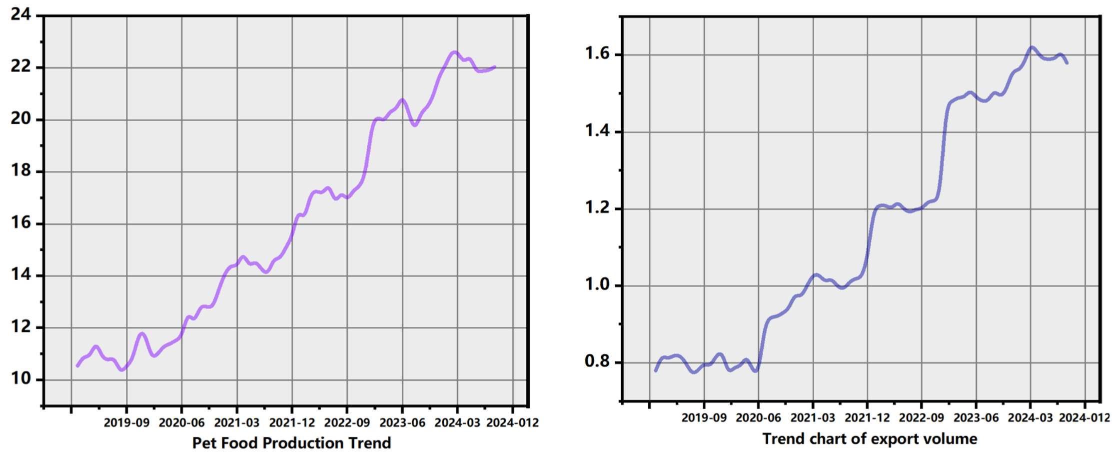

Figure 15 China Pet Food Export Value and Production Trends(2019-2024)

Export Data and Production Data: The figure above presents the export trends of China's pet food industry from 2018 to 2024. Key observations include:

- **Long-Term Growth**: Export values increased steadily, reaching approximately 1.6 billion USD by early 2024.
- **Impact of Global Events**:
  - During the U.S.-China trade tensions (2018–2019), growth stagnated temporarily, reflecting increased trade barriers.
  - The COVID-19 pandemic (2020–2021) caused disruptions in supply chains but led to a surge in global pet ownership, which subsequently boosted demand

for China's exports.

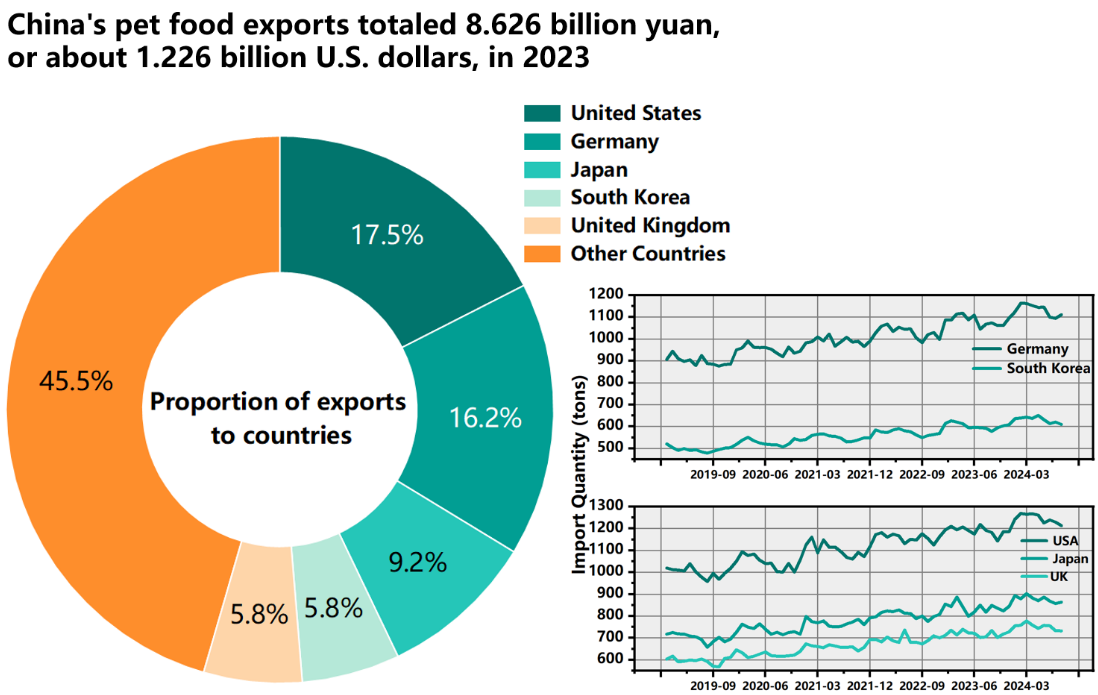

Figure 16 China Pet Food Export Destinations and Export Volume Changes (2019-2024)

Market Distribution: The figure above highlights the market distribution of China's pet food exports in 2023:

- **Dominant Export Markets**:
  - The United States accounted for 45.5% of China's exports, making it the largest trading partner.
  - Germany and Japan each contributed 16.2% and 9.2%, respectively, reflecting strong demand from mature pet care markets.
  - South Korea and the United Kingdom collectively accounted for 11.6%.
- **Other Countries**: Remaining markets accounted for 17.5% of exports, indicating China's growing penetration into diverse global regions.

6.2 The Establishment of Model III

6.2.1 *Analysis of the development of the pet food industry in China*

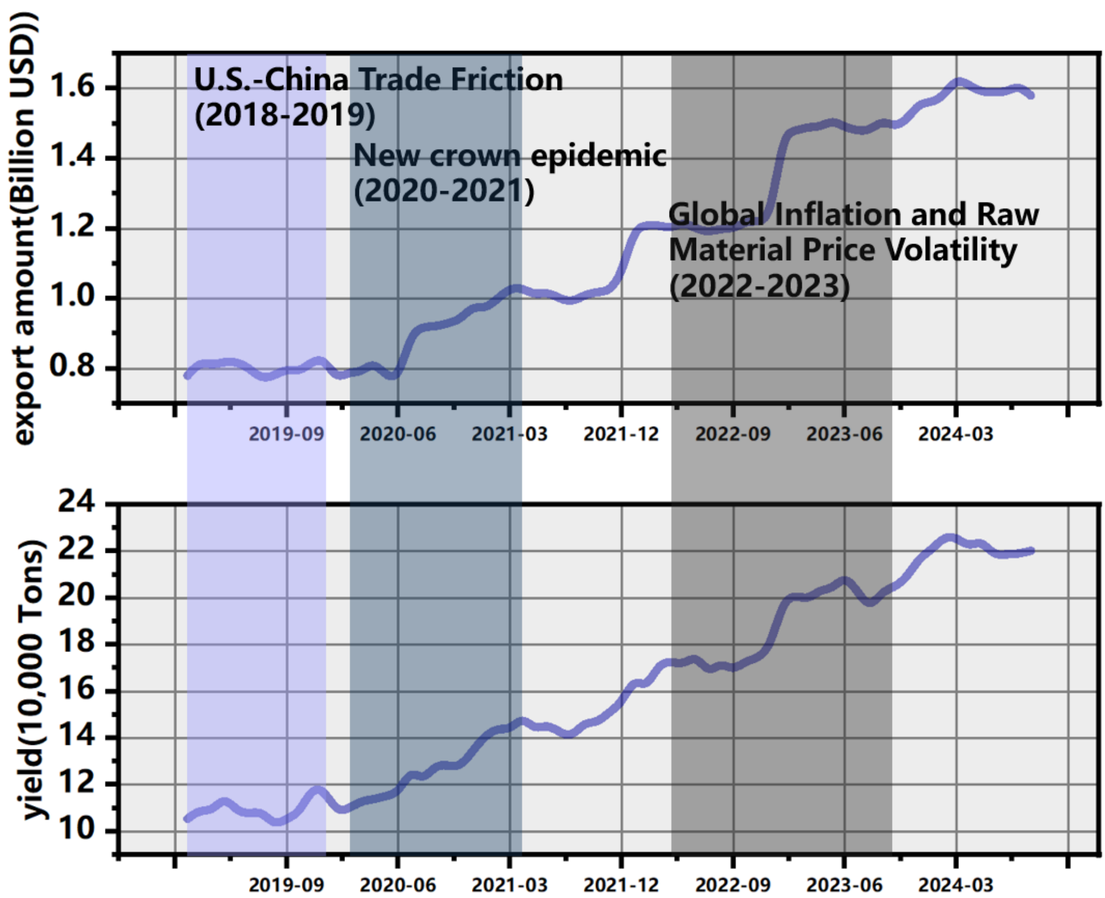

Figure 17 Trend vs. corresponding time module chart

General trend: Growing from about $8billionin2018tonearly16$ billion in 2024, the export value has almost doubled, indicating that the international competitiveness of China's pet food industry continues to grow.

Significant growth points:

- **Post 2020 epidemic**: increased pet ownership globally leads to a surge in demand for pet food, significantly benefiting China as a major exporter.
- **2022-2023**: China's supply chain resilience keeps exports growing despite production cost challenges posed by global inflation and raw material price volatility.

Main export markets :

1. **USA**: Accounting for 45.5% of China's export share, it is the largest importer. This market has a strong demand for high-quality and cost-effective pet food.

2. **Germany and Japan**: 16.2% and 9.2% respectively, dominated by developed markets with increased demand for environmentally friendly and healthy formulated products.
3. **Emerging markets (South Korea, UK, etc.):** significant growth potential despite smaller demand.

6.2.2 *Forecasts on the production and export of pet food in China*

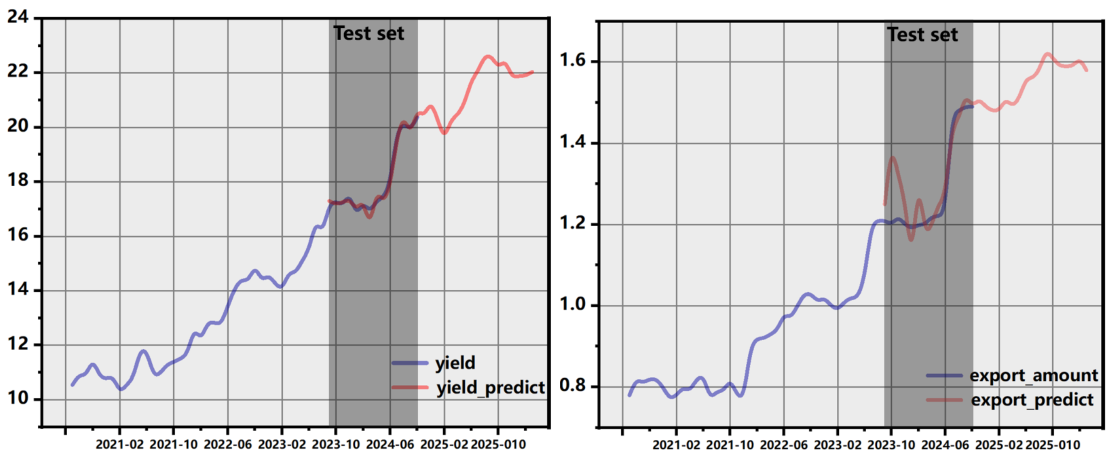

Figure 18 Forecast results of production and export of pet food in China

We input the time series as well as the sales and export vectors into the CNN-LSTM neural network constructed in Problem 2, and obtained the prediction results as shown in Fig.

Historical data and forecast consistency:

- The blue lines indicate historical yields and the red lines are model predictions. In the Test set section, the model's predicted values closely follow the actual values, indicating that the model fits the data well.
- The predicted values are smooth and trend reasonably well, indicating that the model captures both long-term growth trends and short-term fluctuations in the data.

Future Trend Projections:

- From mid-2024 to 2026, production is expected to continue to grow steadily to approximately 240,000 tons (10,000 ton count).
- The growth trend suggests that the industry's production capacity will further increase as the demand for pet food continues to rise.

- Exports will rise steadily from 2024 to 2026, growing from approximately 160$milliontonearly$180 million. This trend is in line with production forecasts, suggesting that export market demand will continue to drive growth in China's pet food industry.

VII. Modeling and solving of Question IV

7.1 Mathematical Modeling for the Sustainable Development of China's Pet Food Industry

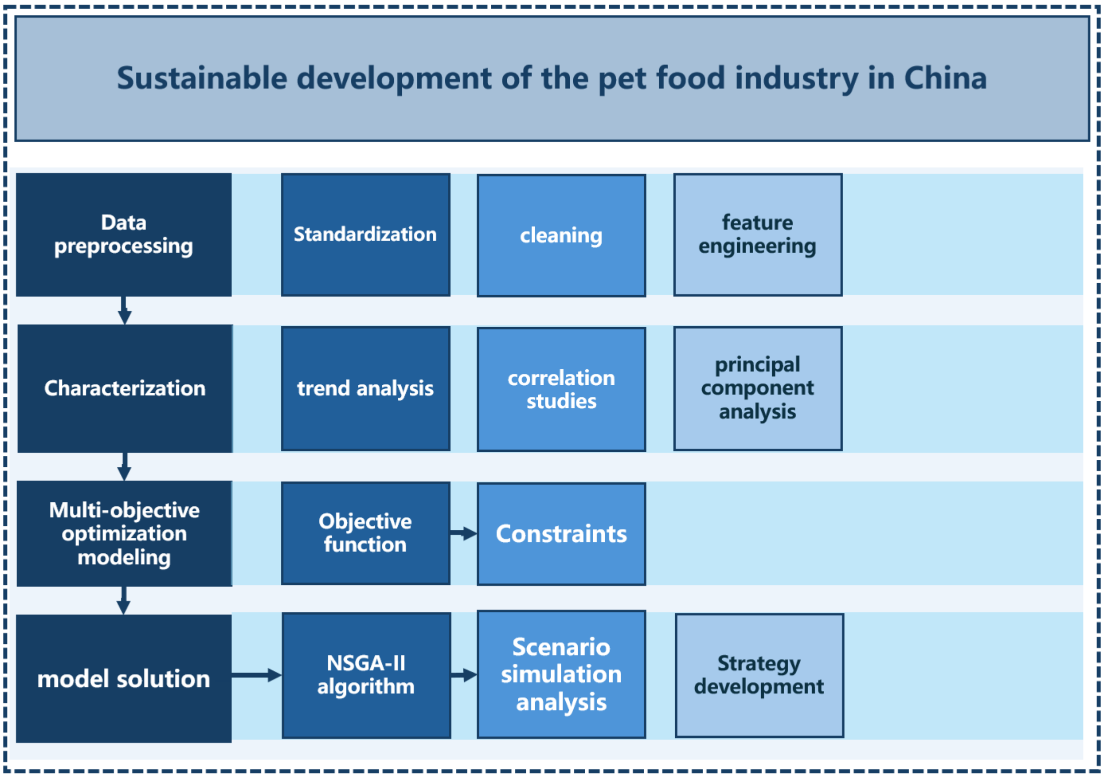

Figure 19 Flowchart of China Pet Food Industry Sustainability Modeling

Data and Indicator System

Data Sources:

- **Historical Data**: Monthly production and export data of China's pet food industry from 2018 to 2024.
- **Macroeconomic Data**: Raw material prices, exchange rates, and inflation rates.
- **Environmental Data**: Carbon emissions and energy consumption per unit of production.

- **International Market Data**: Demand from key export countries such as the United States, Germany, and Japan.

Indicator System: 1. **Economic Indicators**:

- P_t: Monthly production volume at time t (in 10,000 tons).
- E_t: Monthly export value at time t (in billion USD).

2. **Environmental Indicators**:

Carbon emissions:

$$C_t = \beta_c \cdot P_t$$

where β_c is the carbon emission coefficient per unit of production.

Energy consumption:

$$R_t = \beta_r \cdot P_t$$

where β_r is the energy consumption coefficient per unit of production.

3. **Social Indicators:**

Domestic market share:

$$S_t = \frac{P_t - E_t}{\text{Domestic Demand}}$$

The Establishment of Model IV

Objectives:

The study formulates a multi-objective optimization model to balance economic, environmental, and social objectives:

1. **Maximizing Economic Benefits**:

$$Z_1 = \sum_{t=1}^{T} \alpha \cdot P_t + \beta \cdot E_t$$

where α and β are the weights for domestic sales and exports, respectively.

2. **Minimizing Environmental Impact:**

$$Z_2 = \sum_{t=1}^{T} C_t$$

3. **Minimizing Resource Consumption:**

$$Z_3 = \sum_{t=1}^{T} R_t$$

Constraints:

1. **Production Capacity:**

$$P_t \leq \text{Maximum Production Capacity}$$

2. **Carbon Emission Limits:**

$$\sum_{t=1}^{T} C_t \leq \text{Carbon Emission Cap}$$

3. **Export Minimum:**

$$E_t \geq \text{Minimum Export Demand}$$

4. **Non-Negativity:**

$$P_t, E_t \geq 0$$

Solution: Non-Dominated Sorting Genetic Algorithm II (NSGA-II)

Mathematical Explanation of NSGA-II: The NSGA-II algorithm is designed to solve multi-objective optimization problems by finding a Pareto-optimal solution set.

1. **Objective Functions:** Define the objective vector:

$$\mathbf{Z} = (Z_1, Z_2, Z_3)$$

The aim is to simultaneously maximize Z_1 and minimize Z_2 and Z_3.

2. **Non-Dominated Sorting:**

- **Dominance Relationship**:

A solution x^a dominates x^b if and only if:

$$\forall i, f_i(x^a) \leq f_i(x^b) \quad \text{and} \quad \exists j, f_j(x^a) < f_j(x^b)$$

where $f_i(x)$ represents the i-th objective function.

- **Pareto Front:**

The algorithm divides the population into Pareto layers:

- The first layer consists of non-dominated solutions (Pareto-optimal solutions).
- Subsequent layers consist of less optimal solutions.

3. **Crowding Distance:**

To ensure diversity in the Pareto front, the algorithm computes a crowding distance for each solution:

$$d_i = \sum_{j=1}^{M} \frac{f_j^{i+1} - f_j^{i-1}}{f_j^{\max} - f_j^{\min}}$$

where f_j^{i+1} and f_j^{i-1} are the neighboring solutions' objective values for the j-th objective.

4. **Selection, Crossover, and Mutation:**

- **Selection**: Solutions are chosen based on their Pareto rank and crowding distance.
- **Crossover**: Simulated binary crossover (SBX) is applied to generate offspring:

$$c_1 = 0.5 \cdot [(1 + \eta_c) \cdot p_1 + (1 - \eta_c) \cdot p_2]$$

where η_c is the crossover probability.

- **Mutation:**

Mutation introduces diversity by perturbing the offspring:

$$c_i' = c_i + \delta \cdot \mathcal{N}(0, 1)$$

5. **Population Update:**

Combine parent and offspring populations, perform non-dominated sorting, and select the top N solutions for the next generation.

6. **Termination:**

The algorithm stops when the maximum number of generations is reached, or when the Pareto front converges.

VIII. Model Evaluation and Further Discussion

Strengths

The proposed multi-objective optimization model and associated algorithms demonstrate several strengths:

1. **Comprehensive Design**: The model integrates economic, environmental, and social dimensions, ensuring a holistic approach to evaluating the sustainable development of the pet food industry.
2. **Scalability**: The use of NSGA-II allows for scalability in handling multiple objectives and large datasets, making it applicable to other industries and regions.
3. **Dynamic Adaptability**: The inclusion of external factors, such as raw material prices and carbon emission limits, enhances the model's adaptability to real-world changes.
4. **Interpretability**: By leveraging Pareto-optimal solutions, stakeholders can make informed trade-offs between competing objectives, such as maximizing economic growth while minimizing environmental impact.

Weaknesses

Despite its strengths, the model has certain limitations:

1. **Simplified Assumptions**: The linear assumptions in resource consumption and carbon emission calculations may oversimplify the complex relationships between variables.
2. **Limited Data Integration**: The model heavily relies on historical and aggregate data, which may not capture local variations or unexpected disruptions (e.g., policy changes, pandemics).
3. **Computational Complexity**: While NSGA-II is efficient for multi-objective problems, the computation time increases significantly with the number of objectives and constraints.
4. **Uncertainty Handling**: External factors such as policy shifts or sudden price changes are only considered through scenario analysis, lacking a stochastic modeling approach.

Further Discussion

Several avenues exist for future improvement and application: **Advanced Modeling Techniques**:

- Incorporate stochastic programming or robust optimization to handle uncertainties in external factors.
- Use machine learning techniques, such as reinforcement learning, to dynamically adjust strategies based on evolving market conditions.

Integration with Real-Time Data:

- Implement a real-time data integration system to update the model with the latest market, economic, and environmental data.

Geographical Customization:

- Extend the model to consider regional disparities within China, allowing for tailored strategies for different provinces.

Policy Recommendations:

- Use the model outputs to develop specific policy recommendations, such as subsidies for low-carbon technologies or tax incentives for exporters.

By addressing these limitations and incorporating the suggested improvements, the model can become a more robust tool for guiding the sustainable development of the pet food industry in China and beyond.

IX. References

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X. Appendix

Listing 1: Problem 1 solving code

```
import numpy as np
import pandas as pd
from sklearn.preprocessing import MinMaxScaler
from keras.models import Model
from keras.layers import Input, LSTM, Conv1D, MaxPooling1D, Dense,
    Flatten, Concatenate, Attention, Multiply


production_data = data['production'].values
export_data = data['export'].values


scaler = MinMaxScaler(feature_range=(0, 1))
production_scaled = scaler.fit_transform(production_data.reshape(-1, 1))
export_scaled = scaler.fit_transform(export_data.reshape(-1, 1))


def create_dataset(dataset, time_window=12):
    X, y = [], []
    for i in range(len(dataset) - time_window - 1):
        X.append(dataset[i:(i + time_window), 0])
        y.append(dataset[i + time_window, 0])
    return np.array(X), np.array(y)

time_window = 12
X_production, y_production = create_dataset(production_scaled,
    time_window)
X_export, y_export = create_dataset(export_scaled, time_window)


input_production = Input(shape=(time_window, 1))
input_export = Input(shape=(time_window, 1))
```

```
lstm_production = LSTM(64, return_sequences=True)(input_production)
lstm_export = LSTM(64, return_sequences=True)(input_export)


cnn_production = Conv1D(filters=32, kernel_size=3,
    activation='relu')(lstm_production)
cnn_production = MaxPooling1D(pool_size=2)(cnn_production)
cnn_export = Conv1D(filters=32, kernel_size=3,
    activation='relu')(lstm_export)
cnn_export = MaxPooling1D(pool_size=2)(cnn_export)


attention_production = Attention()([lstm_production, lstm_production])
attention_export = Attention()([lstm_export, lstm_export])


flatten_production = Flatten()(cnn_production)
flatten_export = Flatten()(cnn_export)
concat = Concatenate()([flatten_production, flatten_export,
    attention_production, attention_export])


dense1 = Dense(64, activation='relu')(concat)
dense2 = Dense(32, activation='relu')(dense1)
output = Dense(36, activation='linear')(dense2)

model = Model(inputs=[input_production, input_export], outputs=output)
model.compile(optimizer='adam', loss='mse', metrics=['mae'])


train_size = int(len(X_production) * 0.8)
X_train_production, X_test_production = X_production[:train_size],
    X_production[train_size:]
X_train_export, X_test_export = X_export[:train_size],
    X_export[train_size:]
y_train, y_test = y_production[:train_size], y_production[train_size:]
```

```
model.fit([X_train_production, X_train_export], y_train, epochs=50,
    batch_size=32, validation_data=([X_test_production, X_test_export],
    y_test))


predictions = model.predict([X_test_production, X_test_export])


predicted_production = scaler.inverse_transform(predictions)

import matplotlib.pyplot as plt

plt.figure(figsize=(12, 6))
plt.plot(scaler.inverse_transform(y_test.reshape(-1, 1)), label='Actual
    Production')
plt.plot(predicted_production, label='Predicted Production')
plt.legend()
plt.title("China Pet Food Production Prediction")
plt.xlabel("Time")
plt.ylabel("Production")
plt.show()
```